1. (10 points) Multiple Choice. Circle the letter corresponding to the best answer.

(a) In most pumping problems, \( \Delta y \) contributes to which factor in the equation Work = Force \( \times \) Distance?
   A. Force
   B. Distance
   C. Force and Distance
   D. Neither

(b) If we rotate the graph of \( y = f(x) \) around the \( y \)-axis, we usually use
   A. Washers and horizontal rectangles.
   B. Washers and vertical rectangles.
   C. Shells and horizontal rectangles.
   D. Shells and vertical rectangles.

(c) If we rotate the graph of \( x = f(y) \) around the \( y \)-axis, we usually use
   A. Washers and horizontal rectangles.
   B. Washers and vertical rectangles.
   C. Shells and horizontal rectangles.
   D. Shells and vertical rectangles.

(d) Which of the following is the correct definition of \( \ln x \)?
   A. \( \int_0^x \frac{1}{t} \, dt \)
   B. \( \int_1^x \frac{1}{t} \, dt \)
   C. \( \int_0^1 \frac{1}{x} \, dx \)
   D. \( \int_1^t \frac{1}{x} \, dx \)

(e) If \( y = a^x \), then
   A. \( y = e^{x \ln a} \)
   B. \( x = \log_a y \)
   C. \( x = \frac{\ln y}{\ln a} \)
   D. All of the above.
2. (30 points) Compute each integral. Do not use your calculator. Write down all nontrivial substitutions.

(a) \[ \int_0^{\sqrt{\pi}} x \sin(x^2) \cos^2(x^2) \, dx \]

(b) \[ \int_{e^e}^{e^e} \frac{\ln(x)}{x \ln x} \, dx \]

(c) \[ \int \frac{\sec^2 x}{1 + \sqrt{\tan x}} \, dx \]
3. (60 points) Set up integrals or sums of integrals to solve any six of the following seven problems and then use your calculator to evaluate the integrals. Draw pictures, including sample rectangles, washers, shells, etc. Give exact answers (not decimal approximations) unless the problem specifies otherwise.

(a) Find the area of the region bounded by the graphs of \( y = x^3 - 2x \) and \( y = x^2 \).

(b) Find the volume of the solid generated by rotating the region bounded by the graphs of \( y = x + e^x \) and \( y = 1 - x, 0 \leq x \leq 1 \) around the x-axis.

(c) Find the volume of the solid generated by rotating the region bounded by the graphs of \( y = x + e^x \) and \( y = 1 - x, 0 \leq x \leq 1 \) around the y-axis.
(d) Find the volume of the solid generated by rotating the region bounded by the graphs of \( y = x + e^x \) and \( y = 1 - x \), \( 0 \leq x \leq 1 \) around the line \( x = -2 \).

(e) Find the length of the curve given by \( y = x^2 + 1 \) for \( 0 \leq x \leq 2 \). Give an approximate answer.

(f) A force of 5 lbs is required to stretch a spring 4 inches beyond its natural length. Find the work required to stretch it from 6 inches beyond its natural length to 1 foot beyond its natural length. Your integral should compute the answer in ft-lbs.
(g) A tank is in the shape of a cone with a base radius of 20 ft and a height of 40 feet. The vertex of the cone is at the top. If the water in the tank is 30 feet deep, find the work required to pump the water to a point that is 10 feet above the top of the tank. Use 62.5 lbs/ft$^3$ for the density of water.

4. (10 points) Bonus Question. Find the volume of the solid that results from drilling a hole of radius $a$ through the center of a sphere of radius $b$ ($a < b$). Do the integrals without the calculator.

(a) Use the shell method.

(b) Use the washer method.
Part I. Application Problems. You may use your calculator to do integrals. Draw and label pictures, including sample rectangles.

1. (12 points) Find the hydrostatic force on a plate in the shape of an equilateral triangle of side 2 submerged in water as shown below. The bottom is submerged to a depth of 4 feet. Use 62.5 pounds per square inch for the weight density of the water.

2. (12 points) Find the centroid of the region bounded below by $y = x^2 - 4$ and above by $y = 2 - x^2 - 4x$. 